EEN442 - Power Systems II (Συστήματα Ισχύος II)

Part 5: Fault analysis

https://sps.cut.ac.cy/courses/een442/

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This part's learning objectives

After this part of the lecture and additional reading, you should be able to . . .

1. . . . explain the common types of faults occurring in electric power systems;
2. . . . perform fault analysis in simple power systems analytically;
3. . . . use computational tools to perform fault analysis in more complex systems.

Outline

1 Outline

1 Description

- A fault in an electrical power system is the unintentional and undesirable creation of a conducting path (a **short circuit**) or a blockage of current (an **open circuit**)
- This can happen through insulation failure of equipment or flashover of lines initiated by a lightning stroke or through accidental faulty operation
- Depending on the location, the type, the duration, and the system grounding short circuits may lead to:
  - electromagnetic interference with conductors in the vicinity (disturbance of communication lines)
  - stability problems
  - mechanical and thermal stress (i.e. damage of equipment, personal danger)
  - danger for personnel

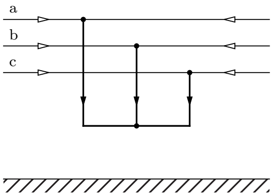
☞ `https://youtu.be/nTWc45JRQGs`

1 Description

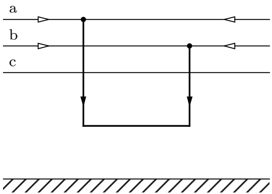
- In high voltage networks, **short circuits** are the most frequent type of faults.
- The system must be **protected** against flow of heavy short circuit currents by disconnecting the faulty part of the system by means of circuit breakers operated by protective relaying
- The safe disconnection can only be guaranteed if the current does not exceed the capability of the circuit breaker
- The short circuit currents in the network must be computed and compared with the ratings of the circuit breakers at regular intervals as part of the normal operation planning

1 Types of short circuits

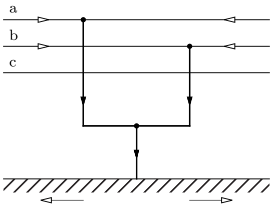
a) Symmetrical three-phase short circuit



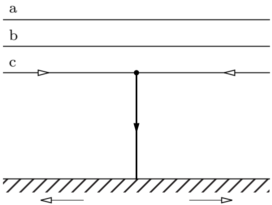
b) Two-phase without ground contact



c) Two-phase with ground contact



d) Single-phase earth fault



SC current

partial SC current in conductor or ground

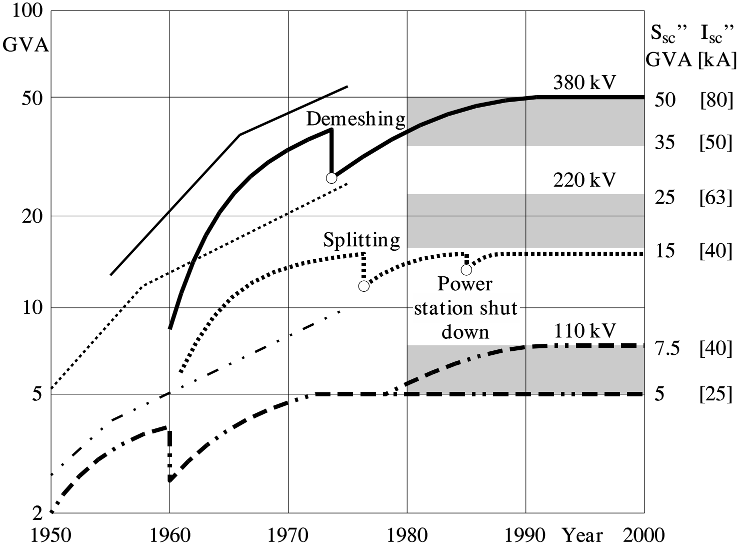
1 Types of short circuits

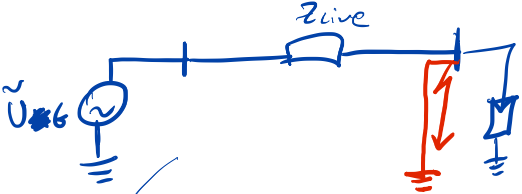
- Short circuits may be solid or may involve an arc impedance.
- The most frequent type of faults are single-phase earth faults, which typically constitute 50 - 80 % of all faults on transmission lines.
- Number of faults vary from region to region and depends on meteorological conditions, e.g. lightning intensity, and other factors.
- In Germany and Switzerland, faults occur with a frequency of 2-5 faults per year and 100 km in the transmission systems.
- In Cyprus, faults occur with a frequency of 1.5-3 faults per year and 100 km in the transmission systems (66kV and above).

1 Types of short circuits

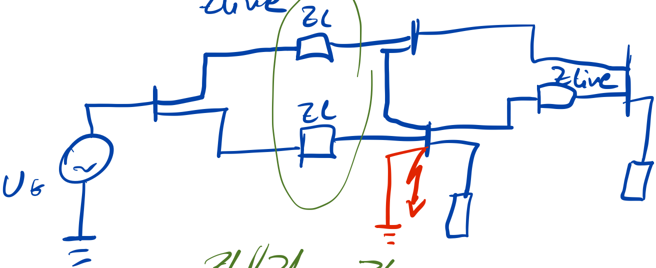
- The majority of system faults **are not** three-phase faults but faults involving one line to ground or occasionally two lines to ground. These are unsymmetrical faults requiring special computational methods like symmetrical components (see previous part of EEN442).
- Symmetrical faults are rare but they lead to the most severe fault current flow against which the system must be protected. They are also simpler to carry out.

1 Development of short circuit currents over the years





$$I_{sc} = \frac{U_G}{Z_{line}}$$

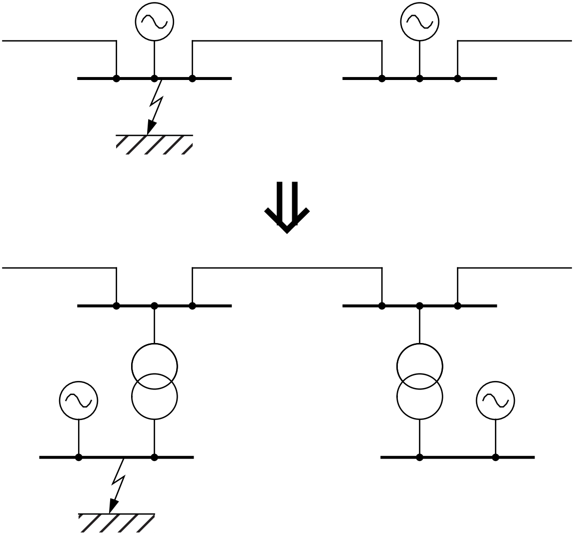


$$Z_L // Z_L = \frac{Z_L}{2}$$

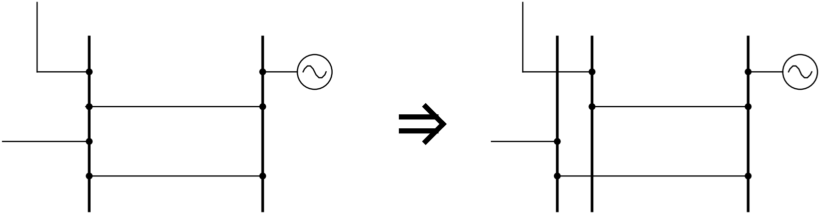
1 Development of short circuit currents over the years

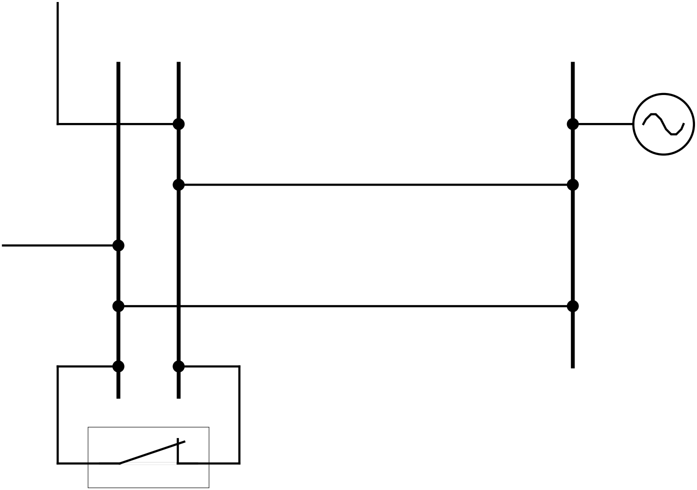
- The short circuit currents at network nodes are generally increasing over the years due to
  - more generators,
  - new lines in existing networks,
  - interconnection of isolated networks to an integrated one.
- This is primarily a problem for the ***expansion planning***, where the impacts of long-term changes on the short circuit currents have to be assessed.
- If the short circuit current exceeds the admissible limit at a network node, the circuit breakers have to be replaced by breakers with higher ratings (**very expensive**)
- Alternatively, the impedance between feeder and fault location can be increased in order to reduce the short circuit current. This is commonly achieved by
  1. introducing a higher voltage level while splitting the existing lower voltage network,
  2. use of multiple busbars,
  3. fast decoupling of busbars during faults.

1 Introduction of a higher voltage level



1 Multiple busbar operation

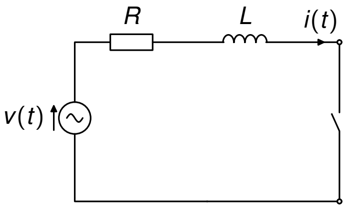




$$t_\mathrm{s} = 70 \ldots 80\mathrm{ms}$$

2 RL circuit

Assume an RL circuit (e.g., the equivalent model of a transmission line) fed by an AC voltage source:



where $v(t) = \sqrt{2} U \sin(\omega t + \alpha)$.

- A short circuit is assumed to take place at $t = 0$. The parameter α controls the instant on the voltage wave when short circuit occurs.
- It is known from circuit theory that the current after short circuit is composed of two parts, i.e.

$$i = i_s + i_t$$

- Assuming the RL complex impedance $\underline{Z}$:

$$\underline{Z} = \sqrt{R^2 + \omega^2 L^2} \angle \left(\theta = \tan^{-1} \frac{\omega L}{R} \right) .$$

- i_s represents the steady state alternating current:

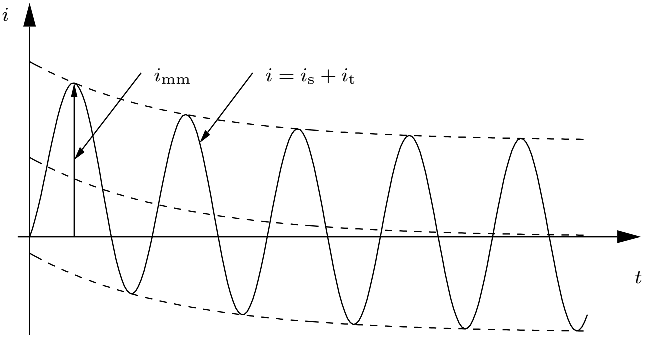
$$i_s = \frac{\sqrt{2}U}{|Z|} \sin(\omega t + \alpha - \theta)$$

- i_t represents the transient direct current:

$$i_t = -i_s(0)\, e^{-(R/L)t} = \frac{\sqrt{2}U}{|Z|} \sin(\theta - \alpha)\, e^{-(R/L)t}$$

2 Steady state and transient direct currents

Plotting the two gives:



- the sinusoidal steady state current is called the ***symmetrical short circuit current***
- the unidirectional transient component is called the ***DC off-set current***
- the total short circuit current to be unsymmetrical until the transient decays

2 Maximum momentary current

- The maximum momentary current that a circuit breaker would need to break at the first peak can be characterized as:

$$i_{\mathrm{mm}} = \frac{\sqrt{2}U}{|Z|}\sin(\theta - \alpha) + \frac{\sqrt{2}U}{|Z|}$$

- Assuming the transmission line resistance is small ($\theta \approx 90^\circ$):

$$i_{\mathrm{mm}} = \frac{\sqrt{2}U}{|Z|}\cos\alpha + \frac{\sqrt{2}U}{|Z|}$$

- The maximum possible value is for $\alpha = 0$, i.e. the short circuit occurring when the voltage wave is going through zero. Thus, i_{mm} may be a high as twice the maximum of the symmetrical short circuit current:

$$i_{\mathrm{mm}} \leq 2\frac{\sqrt{2}U}{|Z|}$$

- For the selection of circuit breakers, momentary short circuit current is taken corresponding to its maximum possible value.

2 Accounting for the DC offset separately

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- Modern circuit breakers are designed to interrupt the current in the first few cycles (five cycles or less)
- When the current is interrupted, the DC off-set i_t has not yet died out and contributes thus to the current to be interrupted
- Computing the value of the DC offset at the time of interruption would be highly complex in a network of even moderately large size. Thus, we:
  1. calculate the initial symmetrical short-circuit current alone using the methods detailed later on
  2. apply an empirical multiplying factor to account for the DC off-set current

2 Accounting for the DC offset separately

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Some of the relevant regulations to account for the DC offset:

- UK Engineering Recommendation ER G7/4:

$$I_{\text{peak}} \approxeq \begin{cases} 1.8\sqrt{2}I''_{\text{sc}} & \text{for low voltage systems } (< 1kV) \\ 2\sqrt{2}I''_{\text{sc}} & \text{for high voltage systems } (> 1kV) \end{cases}$$

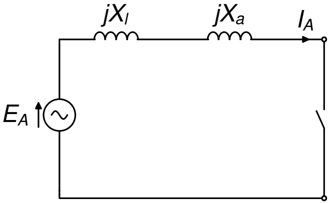
- IEC 909:

$$I_{\text{peak}} \approxeq \sqrt{2}I''_{\text{sc}}(1.02 + 0.98e^{-3(R/X)})$$

  where in radial systems R/X is the ratio of the equivalent short-circuit impedance at the fault location. In meshed systems, it can be:
    - Method A: Uniform Ratio R/X
    - Method B: R/X ratio at short-circuit location
    - Method C: Equivalent frequency

3 Synchronous reactance

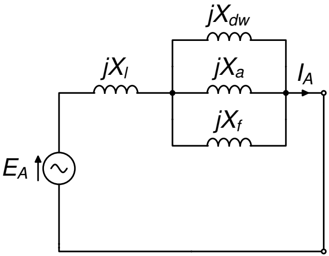
As seen in the previous chapters, the synchronous machine model in steady-state as:



where X_l is the leakage reactance and X_a the armature reactance. Together, they for the ***synchronous reactance*** $X_d = X_a + X_l$.

- In the event of a short circuit, the symmetrical short circuit current is limited initially only by the leakage reactance of the machine.
- Since the air gap flux cannot change instantaneously, to counter the demagnetization of the armature short circuit current, currents appear in the field winding as well as in the damper winding in a direction to help the main flux.
- These currents decay in accordance with the winding time constants. The time constant of the damper winding which has low X/R-ratio is much less than the one of the field winding, which has high leakage inductance with low resistance.

- Thus, during the initial part of the short circuit, the damper and field windings have transformer currents induced in them.
- In the circuit model their reactances—X_{f} of field winding and X_{dw} of damper winding—appear in parallel with X_{a} as shown below



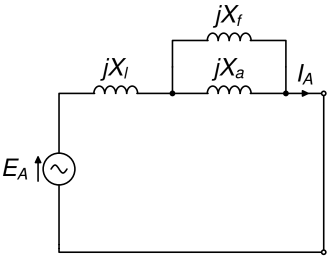
- The total reactance is called ***sub-transient reactance***:

$$X_{\mathrm{d}}'' = X_{\mathrm{l}} + \frac{1}{1/X_{\mathrm{a}} + 1/X_{\mathrm{f}} + 1/X_{\mathrm{dw}}} \tag{3.1}$$

3 Synchronous reactance during short circuit

- As the damper winding currents are first to die out, X_{dw} effectively becomes open circuited.

- The machine reactance thus changes from the parallel combination of X_{a}, X_{f}, and X_{dw} during the initial period of the short circuit to X_{a} and X_{f} in parallel during the middle period.



- The total reactance is called ***transient reactance***:

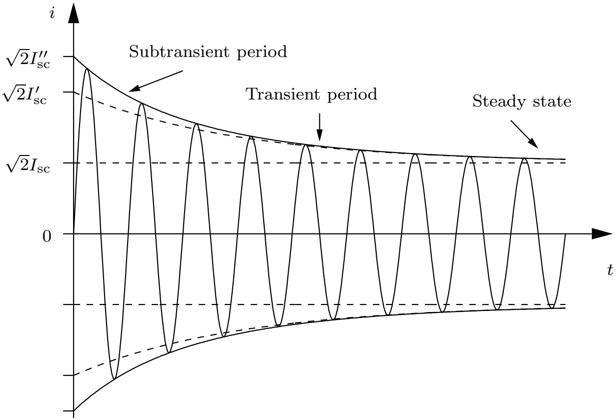
$$X_{\mathrm{d}}' = X_{\mathrm{l}} + \frac{1}{1/X_{\mathrm{a}} + 1/X_{\mathrm{f}}} \tag{3.2}$$

- At a later stage X_{f} dies out and becomes open circuited, returning to the synchronous reactance X_d.

3 Synchronous reactance during short circuit

- Obviously $X''_\mathrm{d} < X'_\mathrm{d} < X_\mathrm{d}$.
- The machine thus offers a time-varying reactance which changes from X''_d to X'_d and finally to X_d.
- Equivalently, during the short-circuit, the current will be split into three different periods:
  - the steady-state current I_sc,
  - the transient current I'_sc, and
  - the initial sub-transient current I''_sc.
- The initial ***sub-transient period*** when the current is large as the machine offers sub-transient reactance
- The middle ***transient period*** where the machine offers transient reactance
- The ***steady-state period*** when the machine offers synchronous reactance

3 Synchronous reactance during short circuit

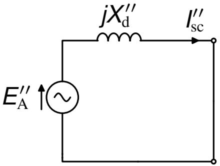


$$I(t) = (I''_{\mathrm{sc}} - I'_{\mathrm{sc}})e^{-\frac{t}{T''_{\mathrm{d}}}} + (I'_{\mathrm{sc}} - I_{\mathrm{sc}})e^{-\frac{t}{T'_{\mathrm{d}}}} + I_{\mathrm{sc}}$$

where T''_{d} and T'_{d} are the machine transient and sub-transient time constants

3 Sub-transient model during short circuit

- In the power flow analysis we are modeling generators as constant power sources (PV characteristics)
- This model is no longer valid during the short-circuit since the power and voltage regulators operate with much larger time constants.
- The modified model below is thus used:

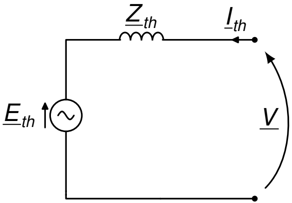


- E''_{A} represents the sub-transient electromagnetic field and X''_{d} the sub-transient reactance of generators or the internal grid impedance of a feeder
- This model may also be applied to large motor loads.

4 Reminder of Thévenin equivalent circuit model

Thévenin's theorem: Seen from one port, a linear circuit can be replaced by an equivalent composed of:

- a voltage source ($\underline{E}_{th}$) = voltage that appears at the port when it is opened
- in series with an impedance ($\underline{Z}_{th} = R_{th} + jX_{th}$) = impedance seen from the port after having removed all sources. For high-voltage systems, R_{th} is frequently ignored.



- Based on the above theorem, when analyzing any load or distribution network connected through one bus to the network, we can represent the higher network with a Thévenin equivalent circuit to simplify the calculations.

4 Short-circuit capacity (or fault level)

- The ***short-circuit capacity (or fault level)*** characterizes both the severity of a fault and the network ”strength” at a given bus

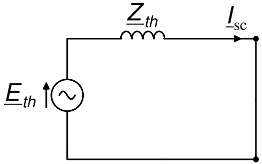
$$S_{sc} = \sqrt{3} V_L I_{sc}$$

where V_L is the line voltage at the bus of the fault and I_{sc} the current in each line of a zero-impedance three-phase short-circuit at the bus

- Used to size circuit breakers
  - the larger I_{sc}, the more ”powerful” the breaker must be to interrupt the electric arc
  - the larger V_L, the larger the recovery voltage at the terminals of the breaker.

4 Relation between short-circuit capacity and Thévenin equivalent

If we want to analyze the short-circuit capacity at a bus where the rest of the network is represented by a Thévenin equivalent.



- The short circuit current is given by

$$|\underline{I}_{\text{sc}}| = \left|\frac{\underline{E}_{th}}{\sqrt{3}\underline{Z}_{th}}\right| = \left|\frac{\underline{V}_{L}}{\sqrt{3}\underline{Z}_{th}}\right|$$

- The short-circuit capacity is given by

$$S_{\text{sc}} = \frac{V_L^2}{|\underline{Z}_{\text{th}}|}$$

- Or, in per-unit with $V_B = V_L$:

$$S_{\text{sc}}^{pu} = \frac{S_{\text{sc}}}{S_B} = \frac{V_L^2}{S_B|\underline{Z}_{\text{th}}|} = \frac{V_B^2}{S_B}\frac{1}{|\underline{Z}_{\text{th}}|} = \frac{Z_B}{|\underline{Z}_{\text{th}}|} = \frac{1}{|\underline{Z}_{\text{th}}^{pu}|}$$

- The larger S_{sc}, the smaller $|\underline{Z}_{\mathrm{th}}|$ and, hence, the smaller the variations of voltage $|\underline{V}_{\mathrm{L}}|$ with changes of the reactive power Q
- Largely fluctuating loads must be connected to buses with large enough S_{sc} values
- As $|\underline{Z}_{\mathrm{th}}| \to 0$: infinite bus

✍ A large S_{sc} requires to install circuit breakers with a strong current interruption capability but it also means that the voltage is better supported when facing disturbances in the network.

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✍ **Definition**: the net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually

Assumptions:

- All line capacitances are ignored.
- Synchronous generators and motors are modeled with the sub-transient model (see 29)
- all loads are represented by constant shunt admittances. Constant power loads are replaced by:

$$Y_L = \frac{P_L - jQ_L}{V_L^2}$$

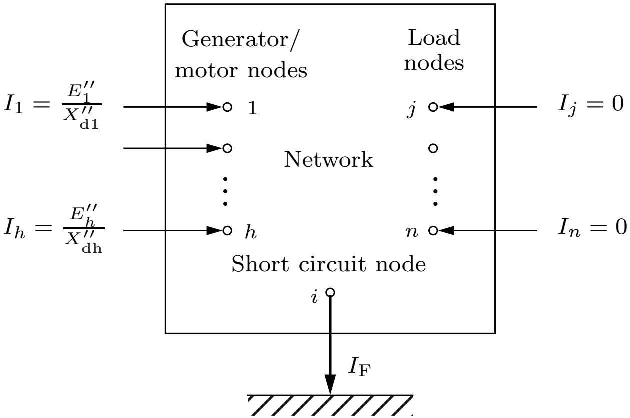
5.1 Short-circuit nodal equations

In order to calculate the short-circuit current I_{F} at node i, we can use the nodal equations

$$\boldsymbol{Y} \cdot \boldsymbol{V} = \boldsymbol{I}$$

- $\boldsymbol{Y}$ is the nodal admittance matrix extended to incorporate the generator sub-transient reactances (X''_{d}).
- $\boldsymbol{V}$ is the nodal voltage vector
- $\boldsymbol{I}$ is the vector of injection currents

$$I_j = \begin{cases} \frac{E''_j}{X''_{\mathrm{d}j}} & \text{for generator nodes} \\ 0 & \text{for load nodes} \\ -I_{\mathrm{F}} & \text{for short-circuit node without generation} \\ -I_{\mathrm{F}} + \frac{E''_j}{X''_{\mathrm{d}j}} & \text{for short-circuit node with generation} \end{cases}$$



Generator/ motor nodes
Load nodes
$I_1 = \frac{E_1''}{X_{d1}''}$
1
j
$I_j = 0$
Network
$I_h = \frac{E_h''}{X_{dh}''}$
h
n
$I_n = 0$
Short circuit node
i
I_F

- Using the symmetrical component theory, the three-phase power system is decomposed in three decoupled networks: positive, negative, and zero-sequence networks.
- Respectively, the fault current is split into I_F^1, I_F^2, and I_F^0.
- Leading to the nodal equations

$$\boldsymbol{Y}^1 \cdot \boldsymbol{V}^1 = \boldsymbol{I}^1, \quad \boldsymbol{Y}^2 \cdot \boldsymbol{V}^2 = \boldsymbol{I}^2, \quad \boldsymbol{Y}^0 \cdot \boldsymbol{V}^0 = \boldsymbol{I}^0$$

- Assuming a fault on node i and no negative or zero-sequence current injections:

$$\boldsymbol{Y}^1 \cdot \begin{pmatrix} \boldsymbol{V}^1_{\neq i} \\ V^1_i \end{pmatrix} = \begin{pmatrix} \boldsymbol{I}^1_{\neq i} \\ -I^1_\mathrm{F} \end{pmatrix}, \quad \boldsymbol{Y}^2 \cdot \begin{pmatrix} \boldsymbol{V}^2_{\neq i} \\ V^2_i \end{pmatrix} = \begin{pmatrix} \boldsymbol{0} \\ -I^2_\mathrm{F} \end{pmatrix}, \quad \boldsymbol{Y}^0 \cdot \begin{pmatrix} \boldsymbol{V}^0_{\neq i} \\ V^0_i \end{pmatrix} = \begin{pmatrix} \boldsymbol{0} \\ -I^0_\mathrm{F} \end{pmatrix}$$

5.1 Short-circuit nodal equations with superposition

- Using the superposition theorem, we can split the nodal equations of the positive sequence to the pre-fault and post-fault:

$$\boldsymbol{Y}^1 \cdot \boldsymbol{V}^1_{pre} = \begin{pmatrix} \boldsymbol{I}^1_{\neq i} \\ 0 \end{pmatrix} \qquad \boldsymbol{Y}^1 \cdot \boldsymbol{V}^1_{post} = \begin{pmatrix} \boldsymbol{I}^1_{\neq i} \\ -I^1_{\mathrm{F}} \end{pmatrix}$$

- Leading to

$$\boldsymbol{Y}^1 \cdot \Delta \boldsymbol{V}^1 = \begin{pmatrix} \boldsymbol{0} \\ -I^1_{\mathrm{F}} \end{pmatrix}$$

where $\Delta \boldsymbol{V}^1 = \boldsymbol{V}^1_{post} - \boldsymbol{V}^1_{pre}$

- Similarly for all symmetrical components:

$$\Delta \boldsymbol{V}^1 = \boldsymbol{Z}^1 \begin{pmatrix} \boldsymbol{0} \\ -I^1_{\mathrm{F}} \end{pmatrix}, \quad \Delta \boldsymbol{V}^2 = \boldsymbol{Z}^2 \begin{pmatrix} \boldsymbol{0} \\ -I^2_{\mathrm{F}} \end{pmatrix}, \quad \Delta \boldsymbol{V}^0 = \boldsymbol{Z}^0 \begin{pmatrix} \boldsymbol{0} \\ -I^0_{\mathrm{F}} \end{pmatrix}$$

where the ***nodal impedance*** matrices $\boldsymbol{Z}^1$, $\boldsymbol{Z}^2$ and $\boldsymbol{Z}^0$, are the inverse matrices of $\boldsymbol{Y}^1$, $\boldsymbol{Y}^2$ and $\boldsymbol{Y}^0$, respectively

5.1 Short-circuit nodal equations with superposition

The previous equation for positive sequence and a fault at node *k* can be expanded to

$$\begin{pmatrix} \Delta V_1^1 \\ \vdots \\ \Delta V_k^1 \\ \vdots \\ \Delta V_N^1 \end{pmatrix} = \begin{pmatrix} Z_{11}^1 & \ddots & Z_{1k}^1 & \ddots & Z_{1N}^1 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{k1}^1 & \ddots & Z_{kk}^1 & \ddots & Z_{kN}^1 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{N1}^1 & \ddots & Z_{Nk}^1 & \ddots & Z_{NN}^1 \end{pmatrix} \begin{pmatrix} 0 \\ \vdots \\ -I_{\mathrm{F}}^1 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} -Z_{1k}^1 I_{\mathrm{F}}^1 \\ \vdots \\ -Z_{kk}^1 I_{\mathrm{F}}^1 \\ \vdots \\ -Z_{Nk}^1 I_{\mathrm{F}}^1 \end{pmatrix} \tag{5.1}$$

Which leads to:

$$\Delta V_k^1 = -Z_{kk}^1 I_{\mathrm{F}}^1$$

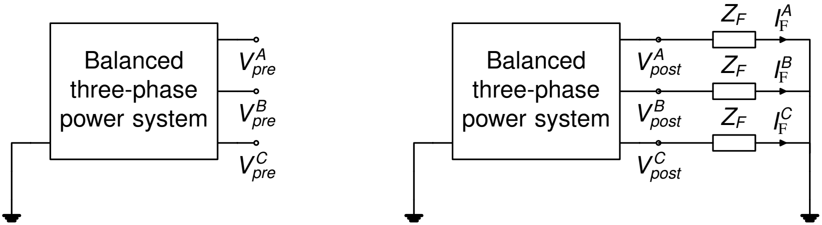
and similarly for the negative and zero sequence:

$$\Delta V_k^2 = -Z_{kk}^2 I_{\mathrm{F}}^2, \quad \Delta V_k^0 = -Z_{kk}^0 I_{\mathrm{F}}^0$$

Expanding when the pre-fault voltages are symmetrical (i.e., $V_{k,pre}^2 = V_{k,pre}^0 = 0$):

$$\boxed{V_{k,post}^1 = V_{k,pre}^1 - Z_{kk}^1 I_{\mathrm{F}}^1, \quad V_{k,post}^2 = -Z_{kk}^2 I_{\mathrm{F}}^2, \quad V_{k,post}^0 = -Z_{kk}^0 I_{\mathrm{F}}^0} \tag{5.2}$$

If we have a three-phase short circuit of impedance Z_F, the diagrams before and after the fault can be depicted as:



The voltages and currents post-fault are:

$$V^A_{post} = Z_F I^A_{\mathrm{F}}, \quad V^B_{post} = Z_F I^B_{\mathrm{F}}, \quad V^C_{post} = Z_F I^C_{\mathrm{F}}, \quad I^A_{\mathrm{F}} + I^B_{\mathrm{F}} + I^C_{\mathrm{F}} = 0$$

The symmetrical components are then:

$$V^1_{post} = V^A_{post}, \quad I^1_{\mathrm{F}} = I^A_{\mathrm{F}}, \quad V^2_{post} = V^0_{post} = I^2_{\mathrm{F}} = I^0_{\mathrm{F}} = 0$$

5.2 Balanced three-phase to earth short-circuit fault

Assuming the short-circuit is on bus k and combining with (5.2) leads to:

$$\boxed{I_{\mathrm{F}}^{1} = \frac{-V_{k,pre}^{1}}{Z_{kk}^{1} + Z_{F}}}, \quad I_{\mathrm{F}}^{2} = 0, \quad I_{\mathrm{F}}^{0} = 0 \tag{5.3}$$

and:

$$I_{\mathrm{F}}^{A} = I_{\mathrm{F}}^{1}, \quad I_{\mathrm{F}}^{B} = \alpha I_{\mathrm{F}}^{A}, \quad I_{\mathrm{F}}^{C} = \alpha^{2} I_{\mathrm{F}}^{A}$$

We can then compute the post-fault voltages from (5.1) as:

$$\begin{pmatrix} V_{1,post}^{1} \\ \vdots \\ V_{k,post}^{1} \\ \vdots \\ V_{N,post}^{1} \end{pmatrix} = \begin{pmatrix} V_{1,pre}^{1} - Z_{1k}^{1} I_{\mathrm{F}}^{1} \\ \vdots \\ V_{k,pre}^{1} - Z_{kk}^{1} I_{\mathrm{F}}^{1} \\ \vdots \\ V_{N,pre}^{1} - Z_{Nk}^{1} I_{\mathrm{F}}^{1} \end{pmatrix} \tag{5.4}$$

and line currents:

$$\boxed{I_{jk,post}^{1} = \frac{V_{j,post}^{1} - V_{k,post}^{1}}{z_{jk}^{1}}}$$

5.2 Balanced three-phase to earth short-circuit fault

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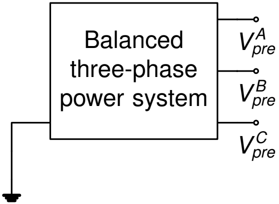
Steps to compute the three-phase to earth short-circuit fault:

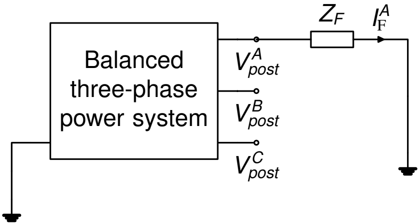
1. Run a power-flow and compute the pre-fault voltages on every node.
2. Convert all power loads into their equivalent impedances (Y_L)
3. Convert all generators and motors to their Norton equivalent using the sub-transient reactances
4. Formulate the positive sequence nodal admittance matrix $\boldsymbol{Y}^1$
5. Compute the nodal impedance matrix $\boldsymbol{Z}^1 = \left(\boldsymbol{Y}^1\right)^{-1}$
6. Compute the fault current using (5.3)
7. Compute the post-fault voltages using (5.4)

✍ A three-phase short-circuit clear of earth can be expressed to a three-phase to earth short-circuit fault using a Δ-Y transformation of the impedances Z_F.

5.2 Single-phase to earth short-circuit fault

If we have a single-phase to ground short-circuit of impedance Z_F, the diagrams before and after the fault can be depicted as:





The voltages and currents post-fault are:

$$V_{post}^{A} = Z_F I_{\mathrm{F}}^{A}, \quad I_{\mathrm{F}}^{B} = I_{\mathrm{F}}^{C} = 0$$

The symmetrical components are then:

$$I_{\mathrm{F}}^{1} = I_{\mathrm{F}}^{2} = I_{\mathrm{F}}^{0} = \frac{1}{3} I_{\mathrm{F}}^{A}$$

Combining the equations in the previous slide and (5.2), we get:

$$\begin{aligned} V^A_{post} = 3Z_F I^1_{\mathrm{F}} &= V^1_{post} + V^2_{post} + V^0_{post} \\ &= V^1_{pre} - Z^1_{kk} I^1_{\mathrm{F}} - Z^2_{kk} I^2_{\mathrm{F}} - Z^0_{kk} I^0_{\mathrm{F}} \\ &= V^1_{pre} - (Z^1_{kk} + Z^2_{kk} + Z^0_{kk}) I^1_{\mathrm{F}} \end{aligned}$$

Leading to the currents:

$$\boxed{I^1_{\mathrm{F}} = I^2_{\mathrm{F}} = I^0_{\mathrm{F}} = \frac{-V^1_{k,pre}}{Z^1_{kk} + Z^2_{kk} + Z^0_{kk} + 3Z_F}} \tag{5.5}$$

Giving:

$$\boxed{I^A_{\mathrm{F}} = 3I^1_{\mathrm{F}}, \quad V^A_{post} = 3Z_F I^1_{\mathrm{F}}} \tag{5.6}$$

5.2 Single-phase to earth short-circuit fault

We can compute the rest of the voltages similar as in (5.4):

$$\begin{pmatrix} V^1_{1,post} \\ \vdots \\ V^1_{k,post} \\ \vdots \\ V^1_{N,post} \end{pmatrix} = \begin{pmatrix} V^1_{1,pre} - Z^1_{1k} I^1_{\mathrm{F}} \\ \vdots \\ V^1_{k,pre} - Z^1_{kk} I^1_{\mathrm{F}} \\ \vdots \\ V^1_{N,pre} - Z^1_{Nk} I^1_{\mathrm{F}} \end{pmatrix}$$

$$\begin{pmatrix} V^2_{1,post} \\ \vdots \\ V^2_{k,post} \\ \vdots \\ V^2_{N,post} \end{pmatrix} = \begin{pmatrix} -Z^2_{1k} I^2_{\mathrm{F}} \\ \vdots \\ -Z^2_{kk} I^2_{\mathrm{F}} \\ \vdots \\ -Z^2_{Nk} I^2_{\mathrm{F}} \end{pmatrix}, \quad \begin{pmatrix} V^0_{1,post} \\ \vdots \\ V^0_{k,post} \\ \vdots \\ V^0_{N,post} \end{pmatrix} = \begin{pmatrix} -Z^0_{1k} I^0_{\mathrm{F}} \\ \vdots \\ -Z^0_{kk} I^0_{\mathrm{F}} \\ \vdots \\ -Z^0_{Nk} I^0_{\mathrm{F}} \end{pmatrix} \tag{5.7}$$

and line currents:

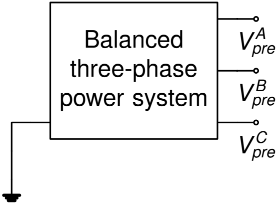
$$I^1_{jk,post} = \frac{V^1_{j,post} - V^1_{k,post}}{z^1_{jk}}, \quad I^2_{jk,post} = \frac{V^2_{j,post} - V^2_{k,post}}{z^2_{jk}}, \quad I^0_{jk,post} = \frac{V^0_{j,post} - V^0_{k,post}}{z^0_{jk}}$$

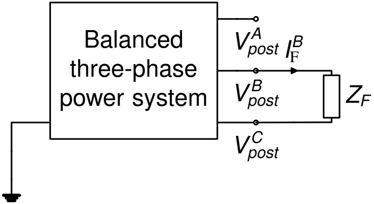
Steps to compute the single-phase to earth short-circuit fault:

1. Run a power-flow and compute the pre-fault voltages on every node.
2. Convert all power loads into their equivalent impedances (Y_L)
3. Convert all generators and motors to their Norton equivalent using the sub-transient reactances
4. Formulate the positive sequence nodal admittance matrices $\boldsymbol{Y}^1$, $\boldsymbol{Y}^2$, and $\boldsymbol{Y}^0$
5. Compute the nodal impedance matrices $\boldsymbol{Z}^1 = \left(\boldsymbol{Y}^1\right)^{-1}$, $\boldsymbol{Z}^2 = \left(\boldsymbol{Y}^2\right)^{-1}$, and $\boldsymbol{Z}^0 = \left(\boldsymbol{Y}^0\right)^{-1}$
6. Compute the fault current using (5.5)-(5.6)
7. Compute the post-fault voltages using (5.7)

5.2 Phase-to-phase or two-phase short-circuit fault

If we have a two-phase short-circuit of impedance Z_F, the diagrams before and after the fault can be depicted as:





The voltages and currents post-fault are:

$$V^B_{post} - V^C_{post} = Z_F I^B_{\mathrm{F}}, \quad I^B_{\mathrm{F}} = -I^C_{\mathrm{F}}, \quad I^A_{\mathrm{F}} = 0$$

The symmetrical components are then:

$$I^1_{\mathrm{F}} = -I^2_{\mathrm{F}}, \quad I^0_{\mathrm{F}} = 0 \xrightarrow{(5.2)} V^0_{post} = 0$$

5.2 Phase-to-phase or two-phase short-circuit fault

Which leads to:

$$\begin{aligned} V^B_{post} - V^C_{post} &= (\alpha^2 V^1_{post} + \alpha V^2_{post}) - (\alpha V^1_{post} + \alpha^2 V^2_{post}) \\ &= (\alpha^2 - \alpha)(V^1_{post} - V^2_{post}) \\ Z_F I^B_{\mathrm{F}} &= Z_F(\alpha^2 I^1_{\mathrm{F}} + \alpha I^2_{\mathrm{F}}) = Z_F(\alpha^2 - \alpha) I^1_{\mathrm{F}} \end{aligned}$$

Equating the two:

$$V^1_{post} - V^2_{post} = Z_F I^1_{\mathrm{F}}$$

Combining with (5.2) for a fault at bus k:

$$V^1_{k,pre} - Z^1_{kk} I^1_{\mathrm{F}} + Z^2_{kk} I^2_{\mathrm{F}} = V^1_{pre} - Z^1_{kk} I^1_{\mathrm{F}} - Z^2_{kk} I^1_{\mathrm{F}} = Z_F I^1_{\mathrm{F}}$$

Leading to:

$$\boxed{I^1_{\mathrm{F}} = -I^2_{\mathrm{F}} = \frac{V^1_{k,pre}}{Z^1_{kk} + Z^2_{kk} + Z_F}} \tag{5.8}$$

The phase currents are then:

$$\boxed{I^B_{\mathrm{F}} = -I^C_{\mathrm{F}} = (\alpha^2 - \alpha) I^1_{\mathrm{F}} = -j\sqrt{3} I^1_{\mathrm{F}}} \tag{5.9}$$

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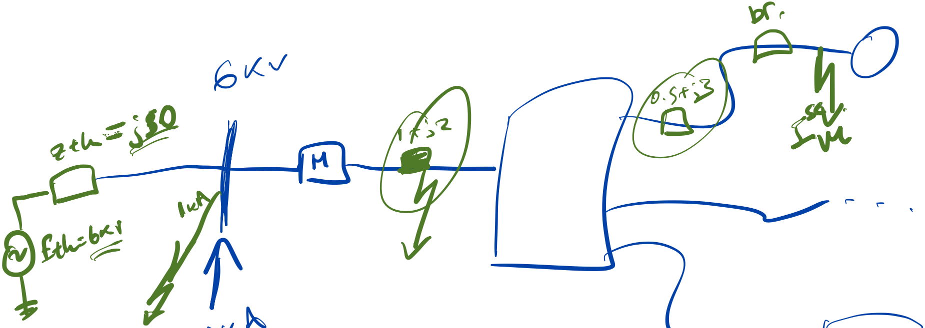
Steps to compute the Phase-to-phase short-circuit fault:

1. Run a power-flow and compute the pre-fault voltages on every node.
2. Convert all power loads into their equivalent impedances (Y_L)
3. Convert all generators and motors to their Norton equivalent using the sub-transient reactances
4. Formulate the positive sequence nodal admittance matrices $\boldsymbol{Y}^1$, $\boldsymbol{Y}^2$
5. Compute the nodal impedance matrices $\boldsymbol{Z}^1 = \left(\boldsymbol{Y}^1\right)^{-1}$, $\boldsymbol{Z}^2 = \left(\boldsymbol{Y}^2\right)^{-1}$
6. Compute the fault current using (5.8)-(5.9)
7. Compute the post-fault voltages using (5.7)



$I_{sc} = 1KA$

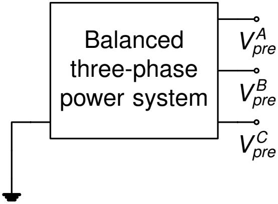
$S_{sc} = 200MVA$

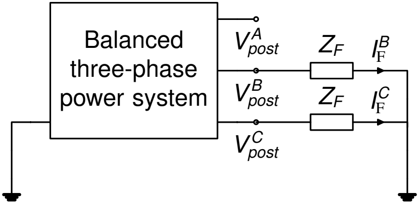
$S_{sc} = \sqrt{3} \cdot 6KV \cdot I_{sc} = 200MVA$

$I_M^{sc} = \dfrac{6KV}{Z_{th} + }$

5.2 Two-phase to earth short-circuit fault

If we have a two-phase short circuit of impedance Z_F to earth, the diagrams before and after the fault can be depicted as:





The voltages and currents post-fault are:

$$V^B_{post} = Z_F I^B_{\mathrm{F}}, \quad V^C_{post} = Z_F I^C_{\mathrm{F}}, \quad I^A_{\mathrm{F}} = 0$$

The symmetrical components are then:

$$I^1_{\mathrm{F}} + I^2_{\mathrm{F}} + I^0_{\mathrm{F}} = 0 \rightarrow I^0_{\mathrm{F}} = -(I^1_{\mathrm{F}} + I^2_{\mathrm{F}})$$

5.2 Two-phase to earth short-circuit fault

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We expand for short-circuit on bus k:

$$V^B_{post} - V^C_{post} = Z_F(I^B_{\mathrm{F}} - I^C_{\mathrm{F}}) \Rightarrow$$
$$V^1_{k,pre} - (Z^1_{kk} + Z_F)I^1_{\mathrm{F}} = -(Z^2_{kk} + Z_F)I^2_{\mathrm{F}}$$

and:

$$V^B_{post} + V^C_{post} = Z_F(I^B_{\mathrm{F}} + I^C_{\mathrm{F}}) \Rightarrow$$
$$-(Z_F + Z^2_{kk})I^2_{\mathrm{F}} = -(Z_F + Z^0_{kk})I^0_{\mathrm{F}} = V^1_{k,pre} - (Z^1_{kk} + Z_F)I^1_{\mathrm{F}}$$

After many calculations[1]:

$$I^1_{\mathrm{F}} = \frac{V^1_{k,pre}[(Z^0_{kk} + Z_F) + (Z^2_{kk} + Z_F)]}{(Z^1_{kk} + Z_F)(Z^2_{kk} + Z_F) + (Z^2_{kk} + Z_F)(Z^0_{kk} + Z_F) + (Z^1_{kk} + Z_F)(Z^0_{kk} + Z_F)}$$
$$I^2_{\mathrm{F}} = \frac{-V^1_{k,pre}(Z^0_{kk} + Z_F)}{(Z^1_{kk} + Z_F)(Z^2_{kk} + Z_F) + (Z^2_{kk} + Z_F)(Z^0_{kk} + Z_F) + (Z^1_{kk} + Z_F)(Z^0_{kk} + Z_F)}$$
$$I^0_{\mathrm{F}} = \frac{-V^1_{k,pre}(Z^2_{kk} + Z_F)}{(Z^1_{kk} + Z_F)(Z^2_{kk} + Z_F) + (Z^2_{kk} + Z_F)(Z^0_{kk} + Z_F) + (Z^1_{kk} + Z_F)(Z^0_{kk} + Z_F)}$$

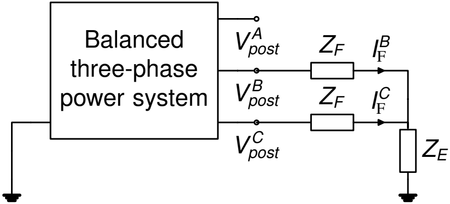
[1] See Chapter 2 of N. Tleis, Power Systems Modelling and Fault Analysis: Theory and Practice, 2nd ed. Academic Press, 2019.

5.2 Two-phase to earth short-circuit fault

The post-fault voltages can be computed by (5.7) and the total current to ground as:

$$I_{\mathrm{F}} = I_{\mathrm{F}}^{B} + I_{\mathrm{F}}^{C} = 3I_{\mathrm{F}}^{0}$$

✍ If an earth impedance Z_E is present in the common connection to earth as shown below, the zero sequence impedance should be updated to $Z_{kk}^0 + 3Z_E$.



5.2 Two-phase to earth short-circuit fault

Steps to compute the single-phase to earth short-circuit fault:

1. Run a power-flow and compute the pre-fault voltages on every node.
2. Convert all power loads into their equivalent impedances (Y_L)
3. Convert all generators and motors to their Norton equivalent using the sub-transient reactances
4. Formulate the positive sequence nodal admittance matrices $\boldsymbol{Y}^1$, $\boldsymbol{Y}^2$, and $\boldsymbol{Y}^0$
5. Compute the nodal impedance matrices $\boldsymbol{Z}^1 = \left(\boldsymbol{Y}^1\right)^{-1}$, $\boldsymbol{Z}^2 = \left(\boldsymbol{Y}^2\right)^{-1}$, and $\boldsymbol{Z}^0 = \left(\boldsymbol{Y}^0\right)^{-1}$
6. Compute the fault current using the equations in Slide 53
7. Compute the post-fault voltages using (5.7)