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EEN452 - Control and Operation of Electric Power Systems Part 6: Power system stability fundamentals https://sps.cut.ac.cy/courses/een452/

Dr Petros Aristidou Department of Electrical Engineering, Computer Engineering & Informatics Last updated: March 16, 2022



After this part of the lecture and additional reading, you should be able to ...

- ... understand the basic classifications of power system stability;
- ② ... be able to identify and perform stability analysis problems; and,
- ③ ... propose methods for stabilizing power systems.

1 Outline



1 Introduction

- 2 Voltage Stability
- **3** Rotor Angle Stability
- 4 References































1 Introduction

2 Voltage Stability

3 Rotor Angle Stability

4 References

2 Voltage stability fundamentals





$$P_l = rac{U_l U_N}{X_e} \sin \phi$$
 $Q_l = rac{U_l U_N \cos \phi - U_l^2}{X_e}$

$$P_{l}^{2} + \left(Q_{l} + \frac{U_{l}^{2}}{X_{e}}\right)^{2} = \left(\frac{U_{l}U_{N}}{X_{e}}\right)^{2} \Rightarrow \left(U_{l}^{2}\right)^{2} + (2Q_{l}X_{e} - U_{N}^{2})U_{l}^{2} + X_{e}^{2}(P_{l}^{2} + Q_{l}^{2}) = 0$$
(2.1)

2 Voltage stability fundamentals



To have (at least) one solution:

$$\left(2Q_lX_e - U_N^2\right)^2 - 4X_e^2\left(P_l^2 + Q_l^2\right) \ge 0 \Rightarrow -\left(\frac{P_lX_e}{U_N}\right)^2 - \frac{Q_lX_e}{U_N^2} + 0.25 \ge 0$$







- any P_i can be reached provided Q_i is adjusted (but U_i may be unacceptable)
- dissymmetry between P_l and Q_l due to reactive transmission impedance
- locus symmetric w.r.t. Q_i axis; this does no longer hold when transmission resistance is included

Under a constant load power factor $\cos \phi$ (i.e., $Q_l = P_l \tan \phi$), we get from Eq. (2.1):

$$\mathcal{P}_l^2+rac{U_N^2}{X_e} an \phi \mathcal{P}_l-rac{U_N^4}{4X_e^2}=0$$

Then, we get the maximum power:

$$P_l^{max} = \frac{\cos\phi}{1+\sin\phi} \frac{U_N^2}{2X_e} \qquad Q_l^{max} = \frac{\sin\phi}{1+\sin\phi} \frac{U_N^2}{2X_e} \qquad U_l^{max} = \frac{U_N}{\sqrt{2}\sqrt{1+\sin\phi}}$$

Or, for the extreme cases:

$$\cos \phi = 1: \qquad P_l^{max} = \frac{U_N^2}{2X_e} \quad Q_l^{max} = 0 \qquad U_l^{max} = \frac{U_N}{\sqrt{2}}$$
$$\cos \phi = 0: \qquad P_l^{max} = 0 \qquad Q_l^{max} = \frac{U_N^2}{4X_e} \qquad U_l^{max} = \frac{U_N}{2}$$

. .0







- for given power (P_l)
 - 1 solution with "high" voltage and "low" current (normal operating point)
 - 1 solution with "low" voltage and "high" current
- compensating the load increases the maximum power but the "critical" voltage approaches normal values
- curves that provide similar information:
 - Q V or S V under constant tan ϕ , Q V under constant P, etc.





- in real systems, much more complicated
 - no infinite bus, voltage control by generators (AVR)
 - multiple loads and generators
 - complex, meshed transmission system with resistive components as well
 - voltage sensitive loads and restorative behavior
 - etc.





- Pre-fault loadability curve of the system (*S*₁)
- Fault occurs in the system
 - 1 Loadability curve is shrunk to Σ_2
 - 2 Post-fault consumption is decreased due to depressed voltages (voltage sensitive loads). If point *B* is outside Σ_2 then there is no solution and we have *short-term voltage instability*
- Loads try to restore consumption to pre-fault point *A* now outside the loadability curve
- Long-term instability leading to voltage collapse





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2 Voltage instability: example (RAMSES with Nordic system)







- Series compensation: very effective but expensive
- Shunt compensation: cheapest mechanism
- SVC and STATCOM devices
- Adjustment of generator active power productions
- Adjustment of generator voltages
- Block load restoration (e.g., through load tap changers) : effective but sometimes too slow
- Undervoltage load shedding : effective but expensive, last resort





3 Outline



1 Introduction

2 Voltage Stability



Rotor Angle Stability

- Transient stability
- Small-disturbance angle stability
- Summary

4 References

- most of the electrical energy today is generated by synchronous machines
- in normal system operation:
 - all synchronous machines rotate at the same electrical speed $\omega_0 = 2\pi f_n$
 - the mechanical and electromagnetic torques acting on the rotating masses of each generator balance each other

 $\dot{\omega}_i = \frac{\omega_0}{2H_i}(T_{mi} - T_{ei})$

- the phase angle differences between the internal e.m.f.'s of the various machines are constant (synchronism)
- following a disturbance, there is an imbalance between the two torques and the rotor speed varies
- rotor angle stability deals with the ability to keep/regain synchronism after being subject to a disturbance





- Transient (angle) stability deals with the ability of the system to keep synchronism after being subject to a **large** disturbance
- typical "large" disturbances:
 - short-circuit cleared by opening of circuit breakers
 - more complex sequences: backup protections, line autoreclosing, etc.
- the nonlinear behavior of the generator and its controllers must be taken into account
 - numerical integration of the differential-algebraic equations is used
- unacceptable consequences of transient instability:
 - generators tripped due to loss of synchronism (to avoid equipment damages)
 - long-lasting voltage dips created by large angle swings (disturb customers)

3.1 Transient (angle) stability





where in steady-state $P_{m0} = P_{e,max} \sin \theta_0$



Multiplying both sides of Eq. (3.1) with $\dot{\theta}$ and integrating:

$$\frac{1}{2}M\dot{\theta}^2 - \int\limits_0^t (P_{m0} - P(\theta))\,\dot{\theta}dt = C$$

Changing the integration variable $(x = \theta(t))$

$$\frac{1}{2}M\dot{\theta}^{2}+\int\limits_{\theta_{0}}^{\theta}\left(P(x)-P_{m0}\right)dx=C$$

"kinetic" energy + "potential" energy = Constant





Туре	symbol	time	
pre-fault	и	<i>t</i> < 0	
during	d	$0 \leq t < t_e$	
post-fault	р	$t \geq t_e$	

θ.

The system is stable if there exists an angle θ_p^i such that the areas are equal ($A_{acc} = A_{dec}$)

Or, for a given
$$\theta_e$$
, $A_{acc} - A_{dec} < 0$

$$A_{acc} = \int_{\theta_u^0}^{\theta_u^j} (P_d(x) - P_{m0}) dx$$
$$A_{dec} = \int_{\theta_e}^{\theta_p^j} (P_p(x) - P_{m0}) dx$$





The system is stable if there exists an angle θ_{ρ}^{i} such that the areas are equal ($A_{acc} = A_{dec}$)

Or, for a given θ_e , $A_{acc} - A_{dec} < 0$

Curves:

- during fault: capability of evacuating power on the network decreased due to low voltages
- post-fault: system weaker owing to the fault clearing actions (e.g., line tripping)

Critical clearing time ($t_e = t_c$):

- Maximum fault duration so that the system returns to equilibrium
- When the system is at the stability limit : $A_{acc} A_{dec} = 0$ and $\theta_e = \theta_c = \theta(t_c)$

3.1 Transient (angle) stability: example (RAMSES with 5-bus system)





Disturbances:

- 6-cycle (120ms) short-circuit without impedance on line "1-3", next to bus 3, cleared by opening that line, when the generator produces 450 MW
- the same fault cleared without line opening, when the generator produces 450 MW
- the same sequence as above, but with the generator producing 400 MW



- Modifying the pre-disturbance operating point:
 - reducing the active power generation
 - operating with higher excitation
- Automatic emergency controls:
 - actions on network: line auto-reclosing, fast series capacitor reinsertion, fast fault clearing single pole breaker operation
 - actions in generators: (turbine) fast valving, generation shedding
 - action on "load": dynamic braking
- Other means:
 - equip generators with fast excitation system
 - control voltage at intermediate points in a long corridor: through synchronous condensers or static var compensators.

3.2 Small-disturbance angle stability

- Cyprus University of Technology
- Small-signal (or small-disturbance) angle stability deals with the ability of the system to keep synchronism after being subject to a "small disturbance"
- "small disturbances" are those for which the system equations can be **linearized** around an equilibrium point
 - tools from linear system theory can be used (in particular eigenvalue and eigenvector analysis)
- following a small disturbance, the variation in electromagnetic torque T_e can be decomposed into:

$$\triangle T_e = K_s \triangle \delta + K_d \triangle \omega$$

 $K_s \triangle \delta$: synchronizing torque $K_d \triangle \omega$: damping torque

- a decrease in synchronizing torque will eventually lead to aperiodic instability (machine "going out of step")
- a decrease in damping torque will eventually lead to oscillatory instability (growing oscillations)





- A small "nudge" to the system (1 ms fault at bus 7) to excite the interarea modes.
- Oscillation of machines 1 and 2 against machines 3 and 4
- Period \sim 2 s





Local modes (involve a small part of the system)

- rotor angle oscillations of a single generator or a single plant against the rest of the system: *local plant mode*
 - can be studied using a one-machine infinite-bus system
- oscillations between rotors of a few generators close to each other: inter-machine or inter-plant mode oscillations
- typical range of frequencies of local plant and inter-plant modes: 0.7 to 2 Hz
- may also be associated with inappropriate tuning of a control equipment (excitation system, HVDC converter, SVC, etc.): control mode



Global modes (involve large areas of the system, widespread effects)

- oscillations of a large group of generators in one area swinging against a group of generators in another area: *interarea mode*
- usually, the larger the group of generators, the slower the oscillations
- typical range of frequencies of interarea modes: 0.1 to 0.7 Hz
- more complex to analyze and to damp

• Let's consider an autonomous system described by the differential equations: $\dot{x} = f(x)$

and
$$\mathbf{x}^*$$
 is an equilibrium point: $\mathbf{f}(\mathbf{x}^*) = \mathbf{0}$

 If we linearize the system around the operating point and ignore higher order terms:

$$\dot{\bigtriangleup x} = \dot{x} = \left. \frac{\partial f}{\partial x} \right|_{x = x^*} x = Ax$$

where $\frac{\partial f}{\partial x}$ is the Jacobian of f with respect to x, and $\mathbf{A} = \frac{\partial f}{\partial x}\Big|_{\mathbf{x}=\mathbf{x}^*}$ is the state matrix of the linearized system.





- Let λ be a real eigenvalue of matrix **A** :
 - $\lambda < 0$: The corresponding mode is stable (decaying exponential).
 - $\lambda > 0$: The corresponding mode is unstable (growing exponential).
 - $\lambda = 0$: The corresponding mode has integrating characteristics.
- Let $\lambda_{1,2} = \sigma \pm j\omega$ be a complex conjugate pair of eigenvalues of **A**:
 - $\Re(\lambda_{1,2}) < 0$: The corresponding mode is stable (decaying oscillation).
 - $\Re(\lambda_{1,2}) > 0$: The corresponding mode is unstable (growing oscillation).
 - \(\mathcal{A}_{1,2}\) = 0: The corresponding mode is critically stable (undamped oscillation).
 - The following dynamic properties can be established:

• Oscillation frequency:
$$f = \frac{\omega}{2\pi}$$

• Damping ratio: $\zeta = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}$

3.2 Dynamic Analysis of the Heffron-Phillips Model







SMIB with classical generator model (mechanical damping torque $\mathcal{K}_D=0$)



Eigenvalues, synchronizing and damping torque coefficients

	$\sigma \pm \mathbf{j}\omega$	ζ	f [Hz]	K _{sync}	K _{damp}
$\lambda_{1,2}$	0 ± <i>j</i> 6.39	-	1.02	0.76	0

Eigenvalues on imaginary axis \longrightarrow system is critically stable



SMIB including field circuit dynamics



Eigenvalues, synchronizing and damping torque coefficients

	$\sigma \pm j\omega$	ζ	f [Hz]	K _{sync}	K _{damp}
$\lambda_{1,2}$	-0.11 ± <i>j</i> 6.41	0.02	1.02	-0.0008	1.53
λ_3	-0.20 ± <i>j</i> 0	1.0	-	-0.77	0

Eigenvalues moved to the left because field circuit adds damping torque



SMIB including excitation system



Eigenvalues, synchronizing and damping torque coefficients

	$\sigma \pm j\omega$	ζ	f [Hz]	K _{sync}	K _{damp}
$\lambda_{1,2}$	0.88 ± <i>j</i> 10.79	-0.08	1.72	0.27	-10.60
λ_3	-33.83 ± <i>j</i> 0	1.0	-	-19.81	0
λ_4	-18.46 ± <i>j</i> 0	1.0	-	-7.01	0

Eigenvalues moved to the right by the excitation system \longrightarrow System is unstable!

Let's assume again the linearized system with the state matrix **A** ($n \times n$) and λ_i is one of its non-zero eigenvalues. Then:

 $Av_i = \lambda_i v_i$

 $\mathbf{A}^T \mathbf{w}_i = \lambda_i \mathbf{w}_i$

• \mathbf{v}_i is the right eigenvector of λ_i :

•
$$\boldsymbol{w}_i$$
 is the left eigenvector of λ_i :

• It can be shown that:
$$W = V^{-1}$$
 and $WAV = diag(\lambda_i) = \Lambda$

$$\boldsymbol{V} = [\boldsymbol{v}_1 \dots \boldsymbol{v}_n] \qquad \boldsymbol{W} = \begin{bmatrix} \boldsymbol{w}_1^T \\ \vdots \\ \boldsymbol{w}_n^T \end{bmatrix}$$





$$\dot{x} = Ax + Bu$$

 $z = Cx + Du$

• We change the variables (y = Wx)

$$\dot{y} = WAVy + WBu = \Lambda y + WBu$$

 $z = CVy + Du$

For the *i*-th "mode" λ_i of the state matrix **A**:
 the larger (**WB**)_i = **w**_i^T**B**, the more the mode can be controlled by **u** the larger (**CV**)_i = **Cv**_i, the more the mode can be observed in **z**



3.2 Transfer function and residues

We can now build the transfer function of the system:

$$H(s) = \frac{Z(s)}{U(s)}$$

= $CV(sI - \Lambda)^{-1}WB + D$
= $[Cv_1 \dots Cv_n] diag(\frac{1}{s - \lambda_i}) \begin{bmatrix} w_1^T B \\ \vdots \\ w_n^T B \end{bmatrix} + D$
= $\sum_{i=1}^n \frac{Cv_i w_i^T B}{s - \lambda_i} + D = \sum_{i=1}^n \frac{R_i}{s - \lambda_i} + D$

The residue R_i relative to the *i*-th mode λ_i :

depends on both the observability and the controllability of the mode

would be zero in case of exact zero-pole cancellation



3.2 Synthesis of a stabilizing feedback using residues





Consider a compensator using *z* as input and acting on *u*:

 Which condition should be satisfied by the transfer function of the PSS in order to stabilize the critical mode λ_c of the uncompensated system?

The closed-loop transfer function is:

 $\frac{H(s)}{1 - K_{PSS}H(s)G_1(s)G_2(s)}$

If \tilde{s} is one of the closed-loop poles:

$$1 - K_{PSS}H(\tilde{s})G_1(\tilde{s})G_2(\tilde{s}) = 0 \Leftrightarrow 1 - K_{PSS}\left[\sum_{i=1}^n \frac{R_i}{\tilde{s} - \lambda_i} + \boldsymbol{D}\right]G_1(\tilde{s})G_2(\tilde{s}) = 0$$

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3.2 Synthesis of a stabilizing feedback using residues





Let's consider a closed-loop pole \tilde{s} lying on the branch of the root locus which starts from the open-loop pole λ_c . When the compensator gain K_{PSS} tends to zero, \tilde{s} tends to λ_c .

Keeping only the dominant terms:

$$1 - R_c G_1(\lambda_c) G_2(\lambda_c) \lim_{K_{PSS} \to 0} \frac{K_{PSS}}{\tilde{s} - \lambda_c} = 0 \Leftrightarrow R_c G_1(\lambda_c) G_2(\lambda_c) = \lim_{K_{PSS} \to 0} \frac{\tilde{s} - \lambda_c}{K_{PSS}}$$

- In the complex plane $\lim_{K_{PSS} \to 0} \frac{\tilde{s} \lambda_c}{K_{PSS}}$ is a vector tangent to the branch of the root locus starting from λ_c .
- In order to shift the eigenvalue to the left:
 - the branch of the root locus should leave λ_c at an angle of 180 degrees. Thus, $G_1(\lambda_c)G_2(\lambda_c)$ must be such that $\angle G_1(j\omega_c)G_2(j\omega_c) = \pm 180 - \angle R_c$
 - $R_c G_1(\lambda_c) G_2(\lambda_c)$ should be a real negative number



Purpose:

Provide additional *damping torque* component in order to prevent the system from becoming unstable.

• Approach:

Insert a feedback between *angular frequency* and *voltage setpoint* to "stabilize" a critical mode λ_c .

Block diagram:



Figure: Block diagram of a simple PSS



Phase Compensation *G*₁:

 shifts λ_c to the left in the complex plane by bringing a phase compensation according to the residue method :

 $\angle G_1(\lambda_c) \simeq \angle G_1(j\omega_c) = \pm 180 - \angle R_c$

- G₁(s)corresponds to one or several lead-lag filters
- the latter are "tuned" to provide their maximum phase shift ϕ_m at the frequency ω_c

Washout Filter G₂:

- in steady state and for slow variations, the PSS must not affect voltage regulation
- G₂(s) is a high-pass filter
- T_w is taken large enough to not modify the phase angle of $G_1(s)$ for frequencies around ω_c . For instance:

$$\frac{10}{T_w} \simeq \frac{\omega_c}{10}$$



Gain K_{PSS}:

• adjusted until the corrected mode $\tilde{\lambda}_c$ has a damping ratio :

$$\zeta = \frac{-\Re(\tilde{\lambda}_c)}{\sqrt{\Re(\tilde{\lambda}_c)^2 + \Im(\tilde{\lambda}_c)^2}} \ge 0.05 - 0.10$$

- while K_{PSS} is increased, the other eigenvalues are monitored since they might move to the right (*the residue method allows controlling a single mode*)
- for excessive values of K_{PSS}, the branch of the root locus that starts from λ_c might "bend" to the right (*the residue method focuses on a neighborhood of the mode to correct*)



Low-pass Filter G₃ (optional):

- in a thermal power plant, the turbine stages, the generator and the exciter are mounted on a relatively long shaft. The latter has torsional oscillation frequencies in the range 10 – 15 Hz and higher
- the PSS must not excite those frequencies
- the risk is higher for a PSS using the rotor speed as input signal
- in this case, *G*₃ is a low-pass filter so that the PSS contribution is negligible at the lowest torsional frequency and above.





3.2 Power System Stabilizer





Cyprus University of Technology

Transient stability:

- depends on operating point and system parameters
- depends also on the disturbance
 - the system may be stable for disturbance 1 but not disturbance 2
 - if so, the system is insecure for 2, but as long as 2 does not happen, it can operate
 - usually, an N-1 security is required

Small-disturbance angle stability:

- depends on operating point and system parameters
- does not depend on the disturbance (assumed infinitesimal and arbitrary)
- is a necessary condition for operating a power system (small disturbances are **always** present)







1 Introduction

- 2 Voltage Stability
- **3** Rotor Angle Stability





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