Comparison of AC Optimal Power Flow Methods in Low-Voltage Distribution Network

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IEEE PES Innovative Smart Grid Technologies Conference ISGT-2021 Europe
18 - 21 October 2021

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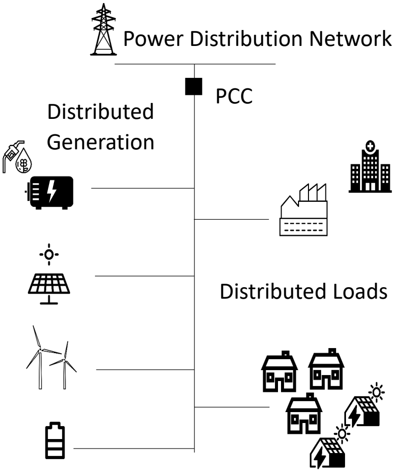
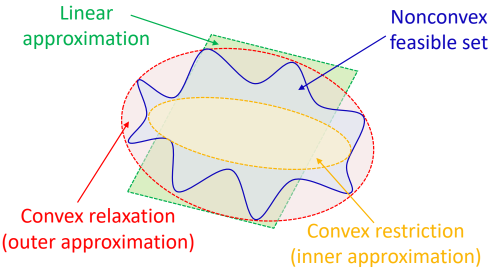
Introduction



- The emergence of Active Distribution Networks and their increased impact on the grid, requires more accurate modeling, both in investment and operation planning problems
- A major tool used for planning in power systems is the Optimal Power Flow (OPF) which aims at obtaining a feasible and optimal operating point that satisfies operational and physical constraints at the minimum cost.
- However, OPF is a complex problem due to the non-linear and non-convex nature of the AC power flow equations that govern the grid's physical laws
- The challenge in finding the solution to an OPF problem, lies between **AC feasibility**, **global optimality**, and **computational efficiency** of the adopted model.

Background and Motivation

- Nonlinear, nonconvex OPF models, provide locally optimal solutions that exactly satisfy power flow equations.
- Convex relaxations/restrictions are tractable alternatives that provide lower/upper bounds on the optimal cost, yield a global optimum and can certify problem feasibility.
- Linear approximations are simplifications to the power flow equations based on assumptions to a certain variable in the network.
- Solutions provided by relaxations, restrictions, and approximations may not be physically applicable in cases leading to AC infeasibility.



Contributions

1. Analysis of five of the most widely adopted OPF formulations used in active distribution networks under different performance metrics i.e. the basic Non-Linear OPF[1], DistFlow (DF)[2], Linearized DistFlow (LinDF)[3] without line shunts, Extended DistFlow (ExDF) with line shunts[4], and Extended Augmented DistFlow (ExAgDF)[5]
2. Comparison of performance in practical situations based on metrics defining the optimality gap and normalized distance to a local AC feasible solution
3. An evaluation of computational performance in a multi-period optimization problem with varying load and generation profiles for the IEEE 34-bus test system, and therefore examine suitability for adoption in LV networks

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[1] Konstantina Christakou et al. "AC OPF in radial distribution networks – Part I: On the limits of the branch flow convexification and the alternating direction method of multipliers". In: Electric Power Systems Research 143 (2017), pp. 438–450. ISSN: 0378-7796.

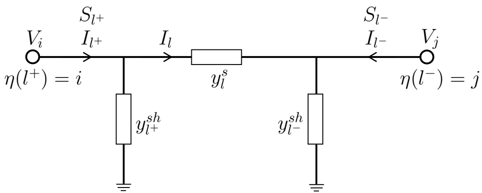
[2] M. Nick et al. "An Exact Convex Formulation of the Optimal Power Flow in Radial Distribution Networks Including Transverse Components". In: IEEE Trans. on Automatic Control 63.3 (2018), pp. 682–697. DOI: 10.1109/TAC.2017.2722100.

[3] M. E. Baran and F. F. Wu. "Network reconfiguration in distribution systems for loss reduction and load balancing". In: IEEE Trans. on Pow. Delivery 4.2 (1989), pp. 1401–1407. DOI: 10.1109/61.25627.

[4] F. Zhou and S. H. Low. "A Note on Branch Flow Models With Line Shunts". In: IEEE Trans. on Pow. Sys. 36.1 (2021), pp. 537–540.

[5] M. Nick et al. "An Exact Convex Formulation of the Optimal Power Flow in Radial Distribution Networks Including Transverse Components". In: IEEE Trans. on Automatic Control 63.3 (2018), pp. 682–697. DOI: 10.1109/TAC.2017.2722100.

Generic AC OPF Formulation



Model 1: Extended AC Optimal Power Flow Model (with line shunts)

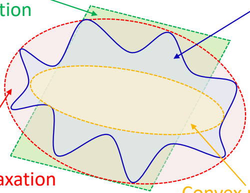
$$S_{l^+} = V_{\eta(l^+)t}(I_{l^+})^*, \qquad S_{l^-} = V_{\eta(l^-)t}(I_{l^-})^*, \qquad \forall lt \quad (1)$$

$$I_{l^+} = y_l^s(V_{\eta(l^+)} - V_{\eta(l^-)}) + y_l^{sh} V_{\eta(l^+)}, \qquad \forall lt \quad (2)$$

$$I_{l^-} = y_l^s(V_{\eta(l^-)} - V_{\eta(l^+)}) + y_l^{sh} V_{\eta(l^-)}, \qquad \forall lt \quad (3)$$

- Power flow equations, (1)-(3), are non-linear resulting in a non- non-convex model only solved through the adoption of non-linear programming (NLP) techniques.
- Model solution converges to local optimality with no guarantees on global optimality.

Approximations and Relaxations to the Branch



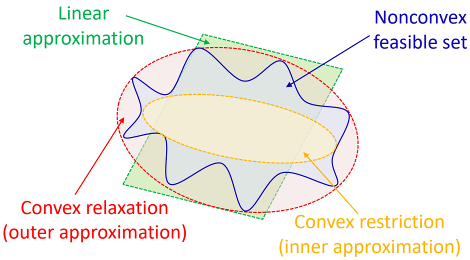
Model 1: Extended AC Optimal Power Flow Model with line shunts (NLP)

- Defined by the nonconvex feasible space

Model 2: Adapted DistFlow Relaxation without line shunts (DF)

- Relaxes the NLP power flow equations based on Second-Order Cone Programming (SOCP)
- Defined by the outer approximation of the feasible space

Approximations and Relaxations to the Branch



Model 1: Extended AC Optimal Power Flow Model with line shunts (NLP)

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Model 2: Adapted DistFlow Relaxation without line shunts (DF)

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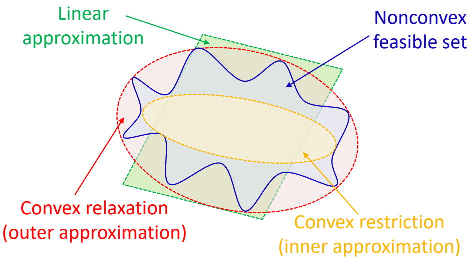
Model 3: Modified Lin-DistFlow Relaxation without line shunts (LinDF)

- Power flow equations defined with the assumption that line losses indicated are negligible in comparison with the active and reactive power flows
- Defined by a linear approximation of the feasible space

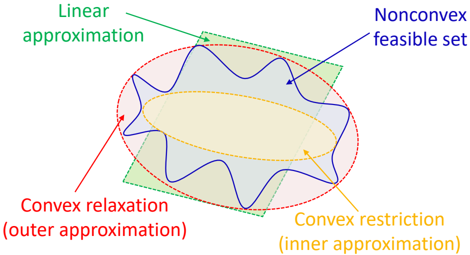
Approximations and Relaxations to the Branch



Model 4: Extended DistFlow Relaxation with Line Shunts (ExDF)

- Current flow here are defined at both ends of the line and not in the longitudinal section
- Defined by the outer approximation of the feasible space based on Second-Order Cone Programming (SOCP)

Approximations and Relaxations to the Branch



Model 4: Extended DistFlow Relaxation with Line Shunts (ExDF)

- Current flow here are defined at both ends of the line and not in the longitudinal section
- Defined by the outer approximation of the feasible space based on Second-Order Cone Programming (SOCP)

Model 5: Augmented DistFlow with Line Shunts (ExAgDF)

- Relaxes the NLP power flow equations based on Second-Order Cone Programming (SOCP)
- Defined by both outer and inner approximations of the feasible space

Model Feasibility Assessment

Optimality Gap

$$\mathrm{OG}^{\mathrm{relax}} = \left| \frac{\Theta^{\mathrm{NLP}} - \Theta^{\mathrm{relax}}}{\Theta^{\mathrm{NLP}}} \right| \tag{4}$$

Average Normalized Deviation

$$\delta_{\chi}^{\mathrm{relax}} = \frac{1}{|\mathcal{T}| \times |\Omega|} \sum_{t \in \mathcal{T}} \sum_{n \in \Omega} \left| \frac{\chi_{nt}^{\mathrm{NLP}} - \chi_{nt}^{\mathrm{relax}}}{\chi_{nt}^{\mathrm{NLP}}} \right| \tag{5}$$

Simulation Results - Optimality Gap and Deviations

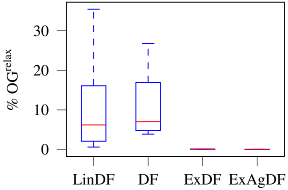


Figure 1: Optimality gap of each model w.r.t the total operational cost of the AC non-linear model solution.

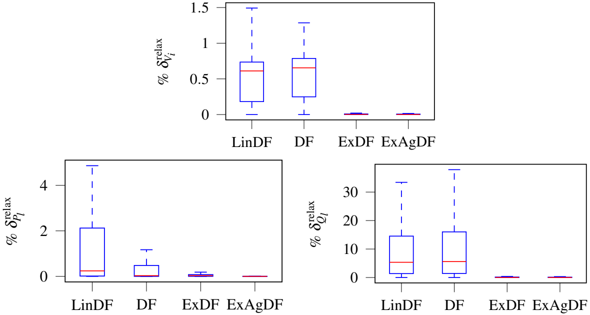


Figure 2: Voltage, active and reactive power flow deviations of the different relaxations to the local solution of the NLP model.

Simulation Results - Computational Performance

Table 1: Computation time, optimal cost and average variations of the different algorithms

	NLP	LinDF	DF	ExDF	ExAgDF
Comput. Time [s]	727.34	0.18	2.04	2.86	171.52
Total Cost [$]	38133	39088	41155	38122	38080
% $\delta_{V_i}^{\mathrm{relax}}$	-	0.52	0.57	0.005	0.003
% $\delta_{p_i}^{\mathrm{relax}}$	-	7.54	3.19	0.24	0.03
% $\delta_{q_i}^{\mathrm{relax}}$	-	23.60	23.65	0.33	0.31
% $\delta_{P_l}^{\mathrm{relax}}$	-	6.69	4.23	0.20	0.03
% $\delta_{Q_l}^{\mathrm{relax}}$	-	14.14	14.58	0.19	0.16

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Conclusions

- The optimality gap metric does not provide a conclusive indication of the feasibility of the power flow approximations and relaxations.
- The divergence of variables in approximated/relaxed models using their average deviations provided an indication of AC feasibility with significant deteriorations where line shunts are ignored.
- Computation time increases with model accuracy thus necessitating a compromise given the size of the study network and end application of the model.

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Thank You!